

A Second Law analysis of the optimum design and operation of thermal energy storage systems

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Abstract—Second Law analysis techniques based on the minimization of entropy generation are applied to the optimal design and operation of a sensible heat thermal energy storage system in which the storage element is both heated and cooled by flowing streams of gases. The results of this study show that (1) an entire operational cycle, which consists of a storage process and a removal process, must be considered (as opposed to the storage process alone) in order to optimize the design and performance of such a system; and (2) a typical optimum system destroys approximately 70–90% of the entering availability and, therefore, has an extremely low thermodynamic efficiency.

INTRODUCTION

ECONOMIES in the design and operation of energy conversion systems often result if some provision is made for the storage of thermal energy. Such provisions must be included in an energy conversion system when the supply of and demand for thermal energy do not coincide in time. Detailed descriptions of thermal energy storage devices for residential-, commercial- and industrial-scale applications appear elsewhere [1–5] and will not be repeated here.

Independent of the specific application of interest, a thermal energy storage system may be characterized by specifying factors such as the physical size of the system, the storage medium employed, and the range of temperatures at which the energy is stored. Of particular importance, however, is the mode of energy storage; that is, whether the energy is stored as sensible energy in a solid or a liquid, as the latent heat required for a phase change of the storage material, or as the heat of reaction in a reversible chemical reaction [6]. Because of their simplicity and their relative cost, sensible heat systems have emerged as the most important class of systems. Therefore, this study is devoted to an analysis of the optimal design and operation of sensible heat thermal energy storage systems.

The classical techniques for the analysis and design of sensible heat energy storage systems are thoroughly described in the excellent book by Schmidt and Willmott [7]. Following the traditional methodology of heat transfer engineers, these techniques are based completely on First Law considerations. From a First Law perspective, the efficiency of a thermal energy storage system can be assessed in terms of how much thermal energy the system can store. Thus, one system is considered to be more effective than another if, for the same energy input in the hot fluid stream entering

the system and the same amount of storage material, it is capable of storing more energy. This approach produces workable designs, *but not necessarily those with the highest possible thermodynamic efficiencies*. It has been shown in recent years that the design of thermodynamically efficient heat transfer equipment must be based on the Second Law of thermodynamics in addition to the First Law. This led Bejan [8] to define an optimum heat transfer system as the least irreversible system that the designer can afford and to develop thermal design techniques based on minimization of entropy generation.

In a pioneering study, Bejan [9] applied his Second Law techniques to the analysis of a sensible heat thermal energy storage system. The power of this analysis resides in Bejan's insight that the primary purpose of a 'thermal energy storage system' is *not*, as the name implies, to store energy, but rather, to store useful work, that is, thermodynamic availability. Thus, his approach is based on minimizing the destruction of thermodynamic availability (entropy generation) as opposed to maximizing the total amount of thermal energy stored. He attempted to obtain the optimum parameters for the design and operation of a thermal energy storage system by examining the energy storage process alone. In practice, however, these systems are operated in a *cyclic* manner, a single cycle consisting of a storage process followed by a removal process. Thus, the present study substantially modifies and extends the Second Law analysis of Bejan to model an entire storage-removal cycle of a sensible heat energy storage system. As will be shown, Bejan's failure to account correctly for the cyclic nature of the operation of a thermal energy storage system leads to erroneous values of the parameters which describe the optimum design and operation of such a system.

ANALYSIS

Description of the system to be analyzed

Consider the sensible heat energy storage system shown in Fig. 1. The system consists of a large liquid bath of mass M and specific heat C contained in a well-insulated vessel. A heat exchanger immersed in the bath allows heat transfer to occur between the gas flows passing through the system and the bath itself. The system operates in a thermodynamic cycle with a single cycle being composed of a storage process followed by a removal process.

During the storage process, valves A and B are open and valves C and D are closed. A constant mass

flowrate, \dot{m}_s , of hot gas at temperature T_{is} and pressure P_{is} enters through valve A, passes through the heat exchanger, and is discharged to the atmosphere through valve B. The bath temperature T and the gas outlet temperature, T_{out} , rise continuously throughout the storage process, both gradually approaching the hot gas inlet temperature, T_{is} . As shown in Fig. 2, the storage process is terminated at time $t = t_s$, when valves A and B are closed and valves C and D are opened to initiate the removal process.

During the removal process, a constant mass flowrate of cold gas, \dot{m}_R , at temperature T_{iR} and pressure P_{iR} enters the system through valve C, passes

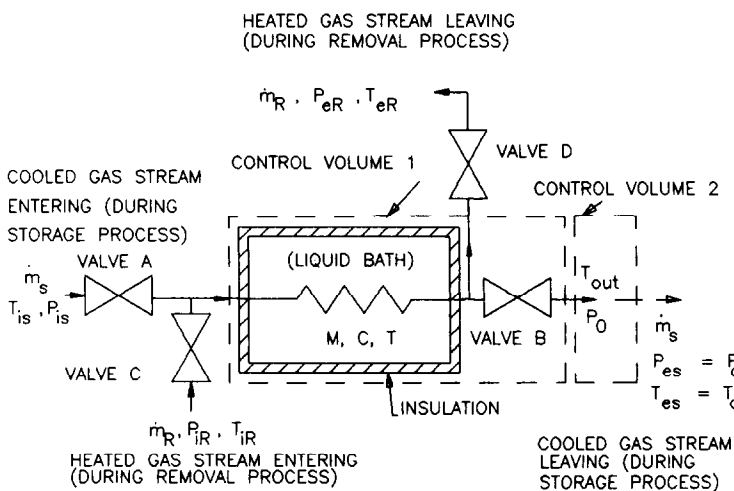


FIG. 1. Schematic of a sensible heat energy storage system.

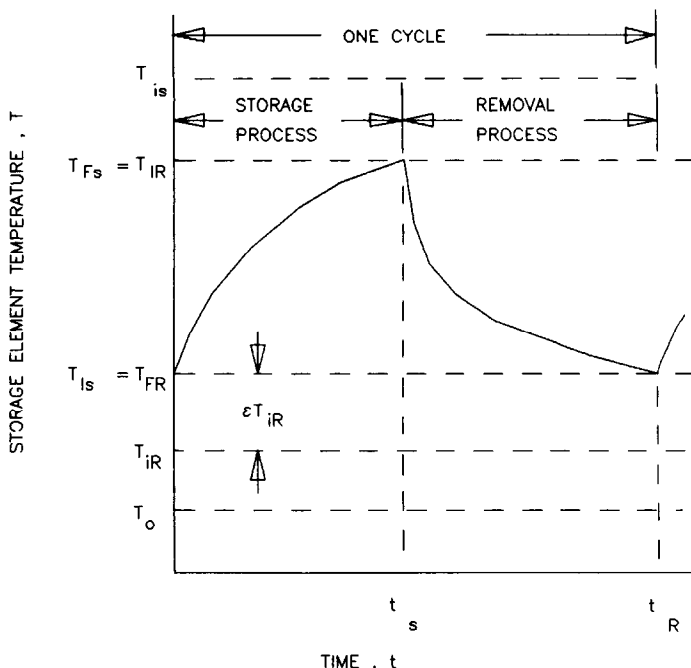


FIG. 2. Storage element temperature history for a complete storage process-removal process cycle.

through the heat exchanger, and exits the system through valve D. This heated flow is not, of course, discharged to the surroundings, but is ducted away for subsequent use in another process. The removal process continues until a time $t = t_R$, when the storage-removal cycle is completed by the bath temperature returning to the same value, $T_{FR} (= T_{IS})$, that it had at the start of the storage process.

Additional assumptions for the analysis

The analysis of the system described in the preceding section is also based on the following assumptions: (a) The bath, which comprises the storage element, is 'well-stirred'. Thus, the temperature is constant throughout the bath at any instant, but can vary with time; that is, the bath is treated as a 'lumped' component. (b) There is no vaporization or condensation in the bath. (c) The same kind of gas flows through the heat exchanger in both the storage and removal processes. This gas is an ideal gas with a constant specific heat, C_p . (d) The storage element temperature at the beginning of the storage process, T_{IS} (= storage element temperature at the end of the removal process, T_{FR}) exceeds the inlet temperature of the heated gas stream, T_{iR} , by an arbitrary amount (εT_{iR}); that is

$$T_{IS} = (1 + \varepsilon)T_{iR} (= T_{FR}) \quad (1)$$

where ε is a positive constant. The parameter ε characterizes the requirement for a thermal energy storage system to have a certain 'tare capacity' in order to deliver thermal energy to the load. (e) There are three sources of entropy generation (irreversibility) in the storage process: (1) heat transfer through the finite temperature difference, $(T_g - T)$, between the gas in the heat exchanger duct and the storage element; (2) heat transfer through the finite temperature difference, $(T_{out} - T_0)$, between the discharged gas and the atmosphere; and (3) fluid friction in the gas flowing through the heat exchanger. (f) There are two sources of entropy generation (irreversibility) in the removal process: (1) heat transfer through the finite temperature difference, $(T_g - T)$, between the gas in the heat exchanger duct and the storage element; and (2) fluid friction in the gas flowing in the heat exchanger duct. (g) The rates of change of energy and entropy stored in the gas in the heat exchanger duct during both the storage and removal processes and of the energy and entropy stored in control volume 2 during the storage process are negligibly small compared to the corresponding rates of change of the energy and entropy stored in the storage element. (h) The overall heat transfer coefficients, U_S and U_R , are each constant along the entire gas flow path in the heat exchanger.

One other source of entropy generation, heat transfer through finite temperature differences *within* the storage element, is not included in the analysis. The inclusion of this effect would yield somewhat lower thermodynamic efficiencies, however, the

present values are adequate first approximations and this assumption allows the considerable analytical simplification afforded by the use of the lumped storage element.

A figure of merit for the design and operation of the system

From the classical, First Law point of view the efficiency of the system should be expressed in terms of the capability of the system to store energy. This results in the well-known expression for the 'First Law efficiency' of a thermal energy storage system given by

$$\eta_I = \frac{\left[\begin{array}{c} \text{energy actually stored in the} \\ \text{bath during the storage period} \end{array} \right]}{\left[\begin{array}{c} \text{maximum possible energy that} \\ \text{could have been stored in the bath} \\ \text{during the same period} \end{array} \right]} \quad (2)$$

Bejan [9], however, has asserted that the commodity of value is not the energy itself, but the thermodynamic availability of this energy. Thus, the appropriate figure of merit based on such Second Law considerations is the ratio of thermodynamic availabilities given by

$$N_S = \frac{\left[\begin{array}{c} \text{'entropy generation number' for} \\ \text{a storage-removal cycle} \end{array} \right]}{\left[\begin{array}{c} \text{total availability destroyed} \\ \text{during the cycle} \end{array} \right]} = \frac{\left[\begin{array}{c} \text{total availability of the cooled} \\ \text{and heated gas streams that} \\ \text{enter the storage unit} \end{array} \right]}{\left[\begin{array}{c} \text{total availability destroyed} \\ \text{during the cycle} \end{array} \right]} \quad (3)$$

When N_S achieves its maximum possible value of unity, all of the availability entering the system is destroyed by irreversibilities. As N_S approaches its lower bound of zero, however, the system approaches completely reversible operation and no availability is destroyed. Therefore, *the system should be designed and operated to minimize the value of N_S* . (This minimum entropy generation criterion was shown by von Spakovsky and Evans [10] to be a special case of the more general optimization approach of Evans [11] and Frangopoulos [12].) This parameter may be placed in a more familiar context by noting that it is related to the so-called 'overall Second Law efficiency' [13]

$$(\eta_{II})_0 = \left[\frac{\text{availability out in product}}{\text{availability in}} \right] \quad (4)$$

by the simple expression

$$N_S = 1 - (\eta_{II})_0 \quad (5)$$

Thus, the method employed in this work may also be regarded as a maximum Second Law efficiency technique.

The total availability of the cooled and heated gas streams that enter the storage unit during a complete cycle may be written as

$$W_{TOTAL} = W_{\Delta P} + W_{\Delta T} \quad (6)$$

where

$$W_{\Delta P} = (W_{Si})_{\Delta P} + (W_{Ri})_{\Delta P} \quad (7a)$$

and

$$W_{\Delta T} = (W_{Si})_{\Delta T} + (W_{Ri})_{\Delta T}. \quad (7b)$$

Invoking the Gouy–Stodola theorem [14], which states that the availability destroyed is proportional to the entropy generated, gives

$$\begin{aligned} & \left[\begin{array}{l} \text{total availability} \\ \text{destroyed during} \\ \text{an entire cycle} \end{array} \right] \\ &= T_0(S_{GEN})_{TOTAL} = T_0[S_{GEN,S} + S_{GEN,R}] \\ &= [T_0][(S_{GEN,S})_{\Delta P} + (S_{GEN,R})_{\Delta P} + (S_{GEN,S})_{\dot{Q}} \\ & \quad + (S_{GEN,S})_{\Delta T} + (S_{GEN,R})_{\Delta T}]. \quad (8) \end{aligned}$$

Substituting equations (7) and (8) in equation (3) and grouping terms such that the contributions of the various sources of entropy generation may be explicitly displayed yields

$$N_S = N_{\Delta P} + N_{\dot{Q}} + N_{S,\Delta T} + N_{R,\Delta T} \quad (9)$$

where

$$N_{\Delta P} = \frac{(S_{GEN,S})_{\Delta P} + (S_{GEN,R})_{\Delta P}}{\bar{W}_{\Delta P} + \bar{W}_{\Delta T}} \quad (10a)$$

$$N_{\dot{Q}} = \frac{(S_{GEN,S})_{\dot{Q}}}{\bar{W}_{\Delta P} + \bar{W}_{\Delta T}} \quad (10b)$$

$$N_{S,\Delta T} = \frac{(S_{GEN,S})_{\Delta T}}{\bar{W}_{\Delta P} + \bar{W}_{\Delta T}} \quad (10c)$$

$$N_{R,\Delta T} = \frac{(S_{GEN,R})_{\Delta T}}{\bar{W}_{\Delta P} + \bar{W}_{\Delta T}}. \quad (10d)$$

Analysis of the storage process

Applying the Second Law and the law of conservation of mass to control volume 1 in Fig. 1 gives for the storage process

$$(S_{GEN,S})_{CV1} = (S_{FS} - S_{IS}) + \dot{m}_S \int_0^{\tau_S} (s_{e1} - s_{i1}) dt. \quad (11)$$

Rewriting the Maxwell relation $T ds = dh - v dP$ for an ideal gas, integrating the resulting expression between the inlet and outlet of control volume 1 and substituting in equation (11) gives

$$\begin{aligned} (S_{GEN,S})_{CV1} &= (S_{FS} - S_{IS}) + \dot{m}_S C_p \int_0^{\tau_S} \ln\left(\frac{T_{out}}{T_{IS}}\right) dt \\ & \quad + \dot{m}_S R \tau_S \ln\left(\frac{P_{IS}}{P_0}\right). \quad (12) \end{aligned}$$

In a similar manner, applying the Second Law and the law of conservation of mass to control volume 2 in Fig. 1 and making use of the Maxwell relation gives for the storage process:

$$\begin{aligned} (S_{GEN,S})_{CV2} &= -\dot{m}_S C_p \int_0^{\tau_S} \ln\left(\frac{T_{out}}{T_0}\right) dt \\ & \quad - \int_0^{\tau_S} \left(\frac{\dot{Q}_{CV2}}{T_0}\right) dt. \quad (13) \end{aligned}$$

Using the First Law and the law of conservation of mass to evaluate the last term in equation (13) yields:

$$\begin{aligned} (S_{GEN,S})_{CV2} &= -\dot{m}_S C_p \int_0^{\tau_S} \ln\left(\frac{T_{out}}{T_0}\right) dt \\ & \quad + \dot{m}_S C_p \int_0^{\tau_S} \left(\frac{T_{out}}{T_0} - 1\right) dt. \quad (14) \end{aligned}$$

Writing the Second Law for the storage element (the liquid bath) gives

$$\frac{dS}{dt} = \frac{\dot{Q}_{SE}}{T} = \frac{MC}{T} \frac{dT}{dt} \quad (15)$$

or

$$\frac{dS}{dt} = \frac{d}{dt} (MC \ln T) \quad (16)$$

where the First Law has been used to evaluate the heat transfer term in equation (15). Integrating equation (16) over the storage period we obtain:

$$(S_{FS} - S_{IS}) = MC \ln\left(\frac{T_{FS}}{T_{IS}}\right). \quad (17)$$

Expressions must now be found for the temperature ratios (T_{out}/T_{IS}) , (T_{out}/T_0) and (T_{FS}/T_{IS}) which appear in equations (12), (14) and (17), respectively. An energy balance on the storage element gives

$$\int_{T_{IS}}^T \frac{dT}{(T_{IS} - T_{out})} = \left(\frac{\dot{m}_S C_p t}{MC}\right) = \theta \quad (18)$$

while the energy balance on a differential length of the heat exchanger duct given by

$$\dot{m}_S C_p dT_g = -U_S(p dx)(T_g - T) \quad (19)$$

may be integrated over the entire length of the duct and shown to yield

$$(T_{IS} - T_{out}) = y_S(T_{IS} - T) \quad (20)$$

where

$$y_S \equiv (1 - e^{-NTU}). \quad (21)$$

Introducing equation (20) into equation (18), integrating and substituting equation (1) in this result, and rewriting in terms of the nondimensional variables gives the following expression for the time-varying temperature of the storage element:

$$T = [1 + \tau_R][T_0] \left[\frac{1 + \tau_S}{1 + \tau_R} + \left(1 + \varepsilon - \frac{1 + \tau_S}{1 + \tau_R}\right) e^{-y_S \theta} \right]. \quad (22)$$

By substituting equation (22) in equation (20), rearranging, and rewriting the results in terms of the nondimensional variables it can easily be shown that

$$\left(\frac{T_{out}}{T_{IS}}\right) = 1 + y_S \left[\frac{(1 + \varepsilon)(1 + \tau_R)}{(1 + \tau_S)} - 1 \right] [e^{-y_S \theta}] \quad (23)$$

and

$$\left(\frac{T_{out}}{T_0}\right) = \{1 + \tau_S\} \left\{ 1 + y_S \left[\frac{(1 + \varepsilon)(1 + \tau_R)}{(1 + \tau_S)} - 1 \right] [e^{-y_S \theta}] \right\}. \quad (24)$$

Finally, after noting that $T = T_{FS}$ when $\theta = \theta_s$, equations (1) and (22) may be used to show that

$$\left(\frac{T_{FS}}{T_{IS}}\right) = \left(\frac{1+\beta}{1+\varepsilon}\right) \quad (25)$$

where

$$\beta \equiv \left[\left(1 + \varepsilon - \frac{1 + \tau_S}{1 + \tau_R}\right) e^{-\gamma_S \theta_S} + \left(\frac{1 + \tau_S}{1 + \tau_R} - 1\right) \right]. \quad (26)$$

The increase of the entropy of the storage element during the storage process may now be found from equations (17) and (25) as

$$(S_{FS} - S_{IS}) = MC \ln\left(\frac{1+\beta}{1+\varepsilon}\right). \quad (27)$$

Then, equations (12), (23) and (37) give

$$\begin{aligned} (S_{GEN,S})_{CV1} = & MC \ln\left(\frac{1+\beta}{1+\varepsilon}\right) \\ & + MC \int_0^{\theta_S} \ln\left\{1 - \left[1 - \frac{(1+\varepsilon)(1+\tau_R)}{(1+\tau_S)}\right]\right. \\ & \times [y_S][e^{-\gamma_S \theta}] \Big\} d\theta \\ & + (MC)\left(\frac{k-1}{k}\right)(\theta_S) \ln\left(\frac{P_{IS}}{P_0}\right) \end{aligned} \quad (28)$$

where the ideal gas relation $R/C_p = (k-1)/k$ has been used in the last term of equation (28). Since the only sources of entropy generation in control volume 1 during the storage process are viscous friction in the heat exchanger duct and heat transfer through the finite temperature difference, $(T_g - T)$, between the gas in the duct and the storage element, equation (28) may be rewritten as

$$(S_{GEN,S})_{CV1} = (MC)(\overline{S_{GEN,S}})_{\Delta P} + (MC)(\overline{S_{GEN,S}})_{\Delta T} \quad (29)$$

where

$$(\overline{S_{GEN,S}})_{\Delta P} = \left(\frac{k-1}{k}\right)(\theta_S) \ln\left(\frac{P_{IS}}{P_0}\right) \quad (30)$$

and

$$\begin{aligned} (\overline{S_{GEN,S}})_{\Delta T} = & \ln\left(\frac{1+\beta}{1+\varepsilon}\right) \\ & + \int_0^{\theta_S} \ln\left\{1 - \left[1 - \frac{(1+\varepsilon)(1+\tau_R)}{(1+\tau_S)}\right]\right. \\ & \times [y_S][e^{-\gamma_S \theta}] \Big\} d\theta. \end{aligned} \quad (31)$$

Similarly, since heat transfer through the finite temperature difference $(T_{out} - T_0)$ between the discharged gas and the surroundings is the only source of entropy generation in control volume 2 during the storage process, equations (14) and (24) yield

$$(S_{GEN,S})_{CV2} = (MC)(\overline{S_{GEN,S}})_{\dot{Q}} \quad (32)$$

where

$$\begin{aligned} (\overline{S_{GEN,S}})_{\dot{Q}} = & [\theta_S][\tau_S - \ln(1 + \tau_S)] + [1 + \tau_R][\varepsilon - \beta] \\ & - \int_0^{\theta_S} \ln\left\{1 - \left[1 - \frac{(1+\varepsilon)(1+\tau_R)}{(1+\tau_S)}\right]\right. \\ & \times [y_S][e^{-\gamma_S \theta}] \Big\} d\theta. \end{aligned} \quad (33)$$

Analysis of the removal process

In a manner identical to that shown for the storage process, it is easily shown for the removal process that

$$\begin{aligned} (S_{GEN,R}) = (S_{GEN,R})_{CV1} = & (MC)(\overline{S_{GEN,R}})_{\Delta P} \\ & + (MC)(\overline{S_{GEN,R}})_{\Delta T} \end{aligned} \quad (34)$$

where

$$\begin{aligned} (\overline{S_{GEN,R}})_{\Delta P} = & \left(\frac{k-1}{k}\right)\left(\frac{\dot{m}_R}{\dot{m}_S}\right)(\theta_R - \theta_S) \ln\left(\frac{P_{IR}}{P_{eR}}\right) \quad (35) \\ (\overline{S_{GEN,R}})_{\Delta T} = & -\ln\left(\frac{1+\beta}{1+\varepsilon}\right) + \left(\frac{\dot{m}_R}{\dot{m}_S}\right) \\ & \times \int_0^{(\theta_R - \theta_S)} \ln[1 + \beta y_R e^{-(\gamma_R)(\dot{m}_R/\dot{m}_S)(\theta - \theta_S)}] d(\theta - \theta_S) \end{aligned} \quad (36)$$

and

$$y_R \equiv [1 - e^{-(U_R/U_S)(\dot{m}_S/\dot{m}_R)(NTU)}]. \quad (37)$$

The integral in equation (36) cannot be solved in closed form. It can, however, be solved numerically in a straightforward manner. In the derivation of the above results energy balances on the heat exchanger duct and control volume 1 gave

$$\frac{T - T_{IR}}{T_{IR} - T_{IR}} = e^{-(\gamma_R)(\dot{m}_R/\dot{m}_S)(\theta - \theta_S)} \quad (38)$$

as an intermediate result, which will be used below.

Completion of the analytical model

First, expressions must be found for the availabilities $W_{\Delta P}$ and $W_{\Delta T}$ that appear in equations (10). For the storage process

$$\begin{aligned} & \left[\begin{array}{l} \text{maximum useful work that could} \\ \text{be performed by the gas that passes} \\ \text{through the heat exchanger} \end{array} \right] \\ & = \left[\begin{array}{l} \text{total mass of} \\ \text{the gas} \end{array} \right] \left[\begin{array}{l} \text{availability per unit} \\ \text{mass of the gas} \\ \text{at inlet conditions} \end{array} \right]. \end{aligned} \quad (39)$$

As shown in classical thermodynamics, the availability of a unit mass of fluid in a steady-state, steady flow process with negligible kinetic and potential energies is given by

$$\Psi = (h - h_0) - T_0(s - s_0). \quad (40)$$

Thus, equation (39) may be rewritten as:

$$W_{Si} = [\dot{m}_S t_S][\dot{m}_S (h_{IS} - h_0) - T_0(s_{IS} - s_0)]. \quad (41)$$

Evaluating equation (41) for an ideal gas and rewriting

in terms of the nondimensional variables gives

$$\bar{W}_{Si} = (W_{Si})_{\Delta P} + (W_{Si})_{\Delta T} \quad (42)$$

where

$$(W_{Si})_{\Delta P} = [MCT_0][\theta_S] \left[\left(\frac{k-1}{k} \right) \ln \left(\frac{P_{iS}}{P_0} \right) \right] \quad (43)$$

and

$$(W_{Si})_{\Delta T} = [MCT_0][\theta_S][\tau_S - \ln(1 + \tau_S)]. \quad (44)$$

Similarly, for the removal process it can be shown that

$$\bar{W}_{Ri} = (W_{Ri})_{\Delta P} + (W_{Ri})_{\Delta T} \quad (45)$$

where

$$(W_{Ri})_{\Delta P} = [MCT_0] \left[\frac{\dot{m}_R}{\dot{m}_S} \right] [\theta_R - \theta_S] \left[\left(\frac{k-1}{k} \right) \ln \left(\frac{P_{iR}}{P_0} \right) \right] \quad (46)$$

and

$$(W_{Ri})_{\Delta T} = [MCT_0] \left[\frac{\dot{m}_R}{\dot{m}_S} \right] [\theta_R - \theta_S] [\tau_R - \ln(1 + \tau_R)]. \quad (47)$$

Thus, equations (7a), (42) and (46), and equations (7b), (44) and (47) give

$$\begin{aligned} \bar{W}_{\Delta P} &= \frac{(W_{Si})_{\Delta P} + (W_{Ri})_{\Delta P}}{(MCT_0)} \\ &= \left[\frac{k-1}{k} \right] \left[\theta_S \ln \left(\frac{P_{iS}}{P_0} \right) + \left(\frac{\dot{m}_R}{\dot{m}_S} \right) (\theta_R - \theta_S) \ln \left(\frac{P_{iR}}{P_0} \right) \right] \end{aligned} \quad (48)$$

and

$$\begin{aligned} \bar{W}_{\Delta T} &= \frac{(W_{Si})_{\Delta T} + (W_{Ri})_{\Delta T}}{(MCT_0)} \\ &= \left\{ [\tau_S - \ln(1 + \tau_S)][\theta_S] \right. \\ &\quad \left. + \left[\frac{\dot{m}_R}{\dot{m}_S} \right] [\theta_R - \theta_S][\tau_R - \ln(1 + \tau_R)] \right\} \end{aligned} \quad (49)$$

respectively.

Expressions must now be found for the pressure ratios (P_{iS}/P_0) and (P_{iR}/P_0) that appear in equations (30), (35) and (48). For the purposes of the present analysis, it will be assumed that the discharge pressure for the removal process is atmospheric. Thus

$$\left(\frac{P_{iR}}{P_{eR}} \right) = \left(\frac{P_{iR}}{P_0} \right). \quad (50)$$

The implication of this assumption is easily seen by using equations (30), (35), (48) and (50) to show that

$$\frac{(\overline{S_{GEN,S}})_{\Delta P} + (\overline{S_{GEN,R}})_{\Delta P}}{\bar{W}_{\Delta P}} = 1. \quad (51)$$

Physically, this result indicates that *all of the available work due to the inlet pressures of the storage and removal gas flows being at greater than atmospheric pressure is completely destroyed by viscous friction*. Next, we make the reasonable assumption that the overall thermal resistance to heat transfer between the gas in the heat exchanger and the storage element is

dominated by the resistance between the gas and the heat exchanger duct wall. Under this assumption the overall heat transfer coefficient is closely approximated by the gas-side heat transfer coefficient such that

$$\left(\frac{U_R}{U_S} \right) = \left(\frac{h_R}{h_S} \right) = \left(\frac{\dot{m}_R}{\dot{m}_S} \right) \left[\frac{(St)_R}{(St)_S} \right] \quad (52)$$

and the parameter y_R may be written as

$$y_R = [1 - e^{-(St)_R/(St)_S NTU}]. \quad (53)$$

It may then be readily shown for the storage process that

$$\left(\frac{P_{iS}}{P_0} \right) = 0.5 + \left\{ 0.25 + \left[\frac{f_S}{8(St)_S} \right] [1 + \tau_S][\bar{G}^2][NTU] \right\}^{1/2} \quad (54)$$

while for the removal process

$$\begin{aligned} \left(\frac{P_{iR}}{P_0} \right) &= 0.5 + \left\{ 0.25 + \left[\frac{f_S}{8(St)_S} \right] \left[\frac{f_R}{f_S} \right] [1 + \tau_R] \right. \\ &\quad \left. \times \left[\frac{\dot{m}_R}{\dot{m}_S} \right]^2 [\bar{G}^2][NTU] \right\}^{1/2}. \end{aligned} \quad (55)$$

To fix ideas, assume that for all of the systems to be analyzed in the present study the flows in the heat exchanger duct may be characterized as turbulent flows in a smooth, round tube such that Reynolds' analogy

$$(St)(Pr)^{2/3} = \left(\frac{f}{8} \right) = (0.023)(Re_{DH})^{-0.2} \quad (56)$$

applies and the Nusselt number is given by [15]:

$$Nu_{DH} = \left(\frac{hD_H}{k} \right) = (0.023)(Re_{DH}^{0.8})(Pr^{1/3}). \quad (57)$$

Introducing equations (56) and (57) into equations (53), (54) and (55) gives

$$y_R = [1 - e^{-(\dot{m}_R/\dot{m}_S)^{-0.2}(NTU)}] \quad (58)$$

$$\left(\frac{P_{iS}}{P_0} \right) = 0.5 + \left\{ 0.25 + [Pr^{2/3}][1 + \tau_S][\bar{G}^2][NTU] \right\}^{1/2} \quad (59)$$

and

$$\begin{aligned} \left(\frac{P_{iR}}{P_0} \right) &= 0.5 + \left\{ 0.25 + [Pr^{2/3}] \left[\frac{\dot{m}_R}{\dot{m}_S} \right]^{1.8} \right. \\ &\quad \left. \times [1 + \tau_R][\bar{G}^2][NTU] \right\}^{1/2} \end{aligned} \quad (60)$$

respectively.

Since the system is operated in a cycle, the dimensionless total cycle time, θ_R , and the dimensionless storage time, θ_S , are not independent of each other. Thus, it is possible to obtain a closed form analytical expression relating these two variables. Noting that $T_{iR} = T_{FS}$, that $T = T_{FR} = T_{iS} = (1 + \varepsilon)(1 + \tau_R)(T_0)$ when $\theta = \theta_R$, substituting

equation (25) in equation (38), and rearranging gives:

$$\theta_R = \theta_S + \ln\left(\frac{\beta}{\varepsilon}\right)^{1/[\gamma_R(\dot{m}_R/\dot{m}_S)]} \quad (61)$$

Thus, the duration of the entire cycle, θ_R , is uniquely determined when the duration of the storage process, θ_S , is specified.

Finally, the First Law efficiency of the system, which is defined in equation (2), may be shown to be given by:

$$\eta_I = 1 - e^{-\gamma_S \theta_S} \quad (62)$$

Summary of the governing equations

The analytical model is composed of 20 equations [equations (9), (10a)–(10d), (21), (26), (30)–(32), (35), (36), (48)–(50), (58)–(62)] in 29 unknowns [N_S , $N_{\Delta P}$, $N_{\dot{Q}}$, $N_{S,\Delta T}$, $N_{R,\Delta T}$, $\bar{W}_{\Delta P}$, $\bar{W}_{\Delta T}$, $(S_{GEN,S})_{\Delta P}$, $(S_{GEN,R})_{\Delta P}$, $(S_{GEN,S})_{\Delta T}$, $(S_{GEN,S})_{\dot{Q}}$, $(S_{GEN,R})_{\Delta T}$, γ_S , γ_R , NTU , β , ε , τ_S , τ_R , θ_S , θ_R , k , Pr , (P_{IS}/P_O) , (P_{IR}/P_{eR}) , (P_{IR}/P_O) , (\dot{m}_R/\dot{m}_S) , \bar{G} , η_I]. Thus, there are nine independent variables in the model.

Description of the optimization study

Numerous design problems can be posed by selecting different sets of the nine independent variables. For the problem examined in the present study, these nine variables were chosen to be τ_S , τ_R , ε , θ_S , (\dot{m}_R/\dot{m}_S) , \bar{G} , NTU , Pr and k . The values of τ_S , τ_R , ε , (\dot{m}_R/\dot{m}_S) , \bar{G} , Pr and k were specified for each system, while optimum values of the dimensionless storage time, $(\theta_S)_{opt}$ (an operational variable), and the number of transfer units, $(NTU)_{opt}$ (a design variable) were computed by an optimization code, GRG2. ('Optimal values' are defined here as those values which minimize the value of the figure of merit, N_S .) This is a realistic design problem which is the counterpart for a complete storage-removal cycle of the problem investigated by Bejan [9] for the storage process alone. Results were obtained for 33 systems, which

represent ranges of the independent variables which include most cases of practical interest. Air was assumed to be the gas used for both the storage and removal processes for all of these systems.

The optimization code, GRG2 [16], is a sophisticated set of routines for solving both linear and nonlinear optimization problems. GRG2 is based on the generalized reduced gradient algorithm [17], which is essentially the upper-bound simplex method of linear programming extended to accommodate a nonlinear objective function.

PRESENTATION AND INTERPRETATION OF RESULTS

Results for a typical optimum system

The results for all 33 systems examined in this study are presented in Tables 1–5. An efficient method of assimilating this information is to begin by focusing attention on the results for a 'typical' optimum system and then to proceed to a more general discussion of the overall results. System No. 15 was selected for this purpose because it is representative of many medium temperature thermal energy storage systems of practical interest. For this system, $\tau_S = 1.0$, $\tau_R = 0.0$, $(\dot{m}_R/\dot{m}_S) = 1.0$, $\bar{G} = 0.05$, $\varepsilon = 0.1$, $k = 1.4$, and $Pr = 0.71$ and the GRG2 optimization program gave $(\theta_S)_{opt} = 0.8634$, $(NTU)_{opt} = 5.5533$, and $(N_S)_{MIN} = 0.7337$.

The dimensionless storage time, $\theta_S = (\dot{m}_S C_p t_S / MC)$, may be interpreted as the ratio of the thermal capacity of the hot gas used in the storage process to the thermal capacity of the storage element. The optimum value of θ_S agrees with Bejan's observation [8] that this parameter should be of order unity for a well-designed system. The number of transfer units (NTU), which is an indicator of heat exchanger size, is reasonable. The optimum value of the entropy generation number (N_S), however, shows that the overall thermodynamic efficiency of this system is

Table 1. Results of optimization study for systems 1–9 ($\tau_S = 1.0$, $\tau_R = 0.0$, $k = 1.4$, $Pr = 0.71$)

Variable	System								
	1	2	3	4	5	6	7	8	9
\dot{m}_R/\dot{m}_S	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
\bar{G}	0.005	0.005	0.005	0.05	0.05	0.05	0.5	0.5	0.5
ε	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$(\theta_S)_{opt}$	1.050	0.9522	0.8313	1.069	0.9660	0.8432	1.469	1.260	1.089
$(NTU)_{opt}$	10.09	10.80	10.00	5.555	5.687	5.744	2.655	2.796	2.866
N_S	0.7399	0.7309	0.7208	0.7490	0.7393	0.7292	0.8751	0.8615	0.8520
θ_R	9.410	6.030	4.441	9.459	6.063	4.470	10.52	6.786	5.004
P_{IS}/P_O	1.0004	1.00006	1.0003	1.021	1.022	1.022	1.643	1.667	1.679
P_{IR}/P_O	1.00005	1.00006	1.00005	1.003	1.003	1.003	1.133	1.140	1.143
$(\eta_{II})_o$	0.2601	0.2690	0.2791	0.2509	0.2606	0.2707	0.1248	0.1384	0.1479
η_I	0.6501	0.6141	0.5645	0.6544	0.6181	0.5685	0.7448	0.6937	0.6419
$N_{\Delta P}$	0.5873×10^{-3}	0.5530×10^{-3}	0.4858×10^{-3}	0.0305	0.0276	0.0264	0.4514	0.4265	0.4153
$N_{\dot{Q}}$	0.2253	0.2199	0.2153	0.2240	0.2182	0.2135	0.1839	0.1766	0.1723
$N_{S,\Delta T}$	0.3071	0.2955	0.2813	0.2945	0.2851	0.2720	0.1344	0.1419	0.1402
$N_{R,\Delta T}$	0.2070	0.2150	0.2237	0.2001	0.2086	0.2173	0.1054	0.1165	0.1243

Table 2. Results of optimization study for systems 10–18 ($\tau_S = 1.0$, $\tau_R = 0.0$, $k = 1.4$, $Pr = 0.71$)

Variable	System								
	10	11	12	13	14	15	16	17	18
\dot{m}_R/\dot{m}_S	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
\bar{G}	0.005	0.005	0.005	0.05	0.05	0.05	0.5	0.5	0.5
ε	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$(\theta_S)_{opt}$	1.051	0.9531	0.8331	1.105	0.9909	0.8634	1.795	1.515	1.309
$(NTU)_{opt}$	9.681	10.28	9.860	5.215	5.447	5.533	2.672	2.814	2.887
N_S	0.7400	0.7310	0.7209	0.7559	0.7445	0.7337	0.9058	0.8894	0.8786
θ_R	5.231	3.492	2.639	5.333	3.560	2.693	6.520	4.426	3.426
P_{IS}/P_0	1.0003	1.0004	1.0003	1.020	1.021	1.021	1.646	1.670	1.682
P_{IR}/P_0	1.0001	1.0002	1.0001	1.010	1.010	1.010	1.384	1.399	1.408
$(\eta_{II})_0$	0.2599	0.2689	0.2790	0.2440	0.2554	0.2662	0.0941	0.1105	0.1213
η_I	0.6504	0.6144	0.5652	0.6669	0.6272	0.5768	0.8119	0.7595	0.7095
$N_{\Delta P}$	0.0011	0.8873×10^{-3}	0.7606×10^{-3}	0.0523	0.0433	0.0396	0.5577	0.5191	0.5000
$N_{\dot{Q}}$	0.2253	0.2201	0.2156	0.2277	0.2209	0.2159	0.1766	0.1742	0.1718
$N_{S,\Delta T}$	0.3068	0.2951	0.2810	0.2807	0.2752	0.2640	0.0904	0.1015	0.1032
$N_{R,\Delta T}$	0.2069	0.2149	0.2236	0.1952	0.2050	0.2142	0.0812	0.0947	0.1037

Table 3. Results of optimization study for systems 19–27 ($\tau_S = 1.0$, $\tau_R = 0.0$, $k = 1.4$, $Pr = 0.71$)

Variable	System								
	19	20	21	22	23	24	25	26	27
\dot{m}_R/\dot{m}_S	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
\bar{G}	0.005	0.005	0.005	0.05	0.05	0.05	0.5	0.5	0.5
ε	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$(\theta_S)_{opt}$	1.053	0.9543	0.8338	1.200	1.058	0.9198	2.131	1.798	1.564
$(NTU)_{opt}$	9.267	9.698	9.885	4.678	5.001	5.118	3.055	3.152	3.208
N_S	0.7405	0.7313	0.7212	0.7741	0.7587	0.7463	0.9326	0.9164	0.9056
θ_R	3.144	2.224	1.737	3.360	2.371	1.858	4.532	3.299	2.671
P_{IS}/P_0	1.0003	1.0003	1.0003	1.018	1.019	1.019	1.710	1.726	1.735
P_{IR}/P_0	1.0006	1.0006	1.0006	1.031	1.033	1.034	2.038	2.060	2.072
$(\eta_{II})_0$	0.2594	0.2686	0.2787	0.2258	0.2412	0.2536	0.0673	0.0835	0.0943
η_I	0.6512	0.6149	0.5655	0.6954	0.6505	0.5992	0.8688	0.8212	0.7771
$N_{\Delta P}$	0.0027	0.0020	0.0017	0.1076	0.0861	0.0762	0.6660	0.6201	0.5957
$N_{\dot{Q}}$	0.2256	0.2201	0.2156	0.2358	0.2271	0.2216	0.1523	0.1576	0.1591
$N_{S,\Delta T}$	0.3057	0.2945	0.2805	0.2478	0.2502	0.2429	0.0570	0.0678	0.0706
$N_{R,\Delta T}$	0.2065	0.2147	0.2234	0.1830	0.1954	0.2057	0.0573	0.0710	0.0801

Table 4. Results of optimization study for systems 28–30 ($\tau_S = 2.0$, $\tau_R = 0.0$, $k = 1.4$, $Pr = 0.71$)

Variable	System		
	28	29	30
\dot{m}_R/\dot{m}_S	1.0	1.0	1.0
\bar{G}	0.05	0.05	0.05
ε	0.01	0.05	0.1
$(\theta_S)_{opt}$	0.9859	0.9268	0.8608
$(NTU)_{opt}$	5.913	6.121	6.217
N_S	0.7141	0.7069	0.6996
θ_R	5.831	4.134	3.347
P_{IS}/P_0	1.034	1.035	1.036
P_{IR}/P_0	1.012	1.012	1.012
$(\eta_{II})_0$	0.2859	0.2931	0.3004
η_I	0.6259	0.6034	0.5765
$N_{\Delta P}$	0.0279	0.0235	0.0218
$N_{\dot{Q}}$	0.2240	0.2195	0.2155
$N_{S,\Delta T}$	0.2645	0.2603	0.2531
$N_{R,\Delta T}$	0.1977	0.2035	0.2093

Table 5. Results of optimization study for systems 31–33 ($\tau_S = 2.0$, $\tau_R = 0.1$, $k = 1.4$, $Pr = 0.71$)

Variable	System		
	31	32	33
\dot{m}_R/\dot{m}_S	1.0	1.0	1.0
\bar{G}	0.05	0.05	0.05
ε	0.01	0.05	0.1
$(\theta_S)_{opt}$	0.6499	0.6224	0.5681
$(NTU)_{opt}$	5.425	5.728	5.864
N_S	0.6334	0.6325	0.6284
θ_R	5.086	3.435	2.658
P_{IS}/P_0	1.031	1.033	1.034
P_{IR}/P_0	1.012	1.012	1.013
$(\eta_{II})_0$	0.3666	0.3675	0.3716
η_I	0.4764	0.4623	0.4325
$N_{\Delta P}$	0.0327	0.0266	0.0242
$N_{\dot{Q}}$	0.1545	0.1615	0.1634
$N_{S,\Delta T}$	0.2907	0.2812	0.2714
$N_{R,\Delta T}$	0.1554	0.1632	0.1695

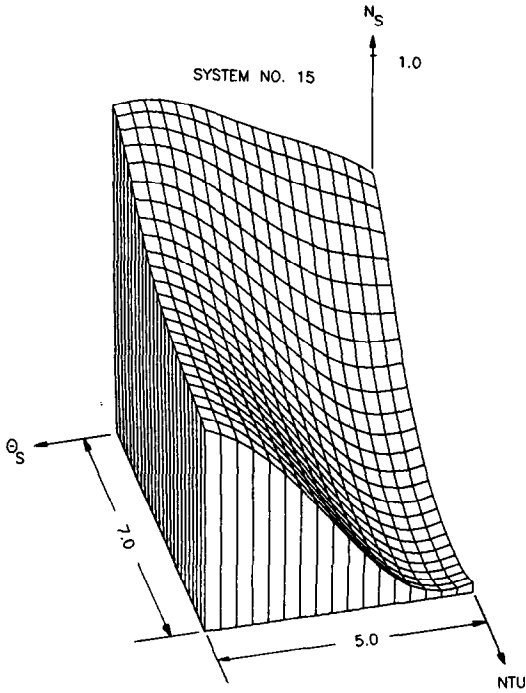


FIG. 3. Typical entropy generation number surface.

extremely low. Since $\tau_R = 0.0$ and the pressure drops across the system are quite small, almost all ($\approx 96\%$) of the total thermodynamic availability of the storage and removal gas flows is concentrated in the availability of the storage gas flow due to the fact that it is at a temperature higher than that of the surroundings. An entropy generation number of

0.7337 means that 73.37% of this availability is destroyed during a storage-removal cycle. Thus, only 26.63% of this availability is delivered in the exiting stream of heated gas during the removal process. (The Second Law efficiency is only 26.63%.)

The entropy generation number surface [$N_S = N_S(\theta_S, NTU)$] is shown in Fig. 3. This surface is representative of the surfaces for all the systems examined in this study. Visual inspection shows that there is no local minimum point that could be mistakenly identified as a global minimum by the optimization program. A detailed examination of the data used to plot this surface indicates that the program has correctly located the global minimum point of N_S .

The curve formed by the intersection of the entropy generation number surface with the plane for which $NTU = (NTU)_{opt}$ is shown in Fig. 4. As expected, this curve indicates that N_S always increases when the storage time deviates from its optimum value of 0.8634. The relative contributions of all the sources of irreversibility in the system to the entropy generation number, N_S , which are also shown in Fig. 4, offer some insight into this behavior. In the limit of $\theta_S \rightarrow 0$, the largest source of entropy generation is heat transfer through finite temperature differences in the heat exchanger duct during the storage process, which is represented by $N_{S,\Delta T}$. This source, however, rapidly diminishes in importance with increasing time until, at the optimum storage time of 0.8634, it contributes 36% of the total entropy generated. In the limit of $\theta_S \rightarrow \infty$, this source of irreversibility continues to decrease in importance and heat transfer between the

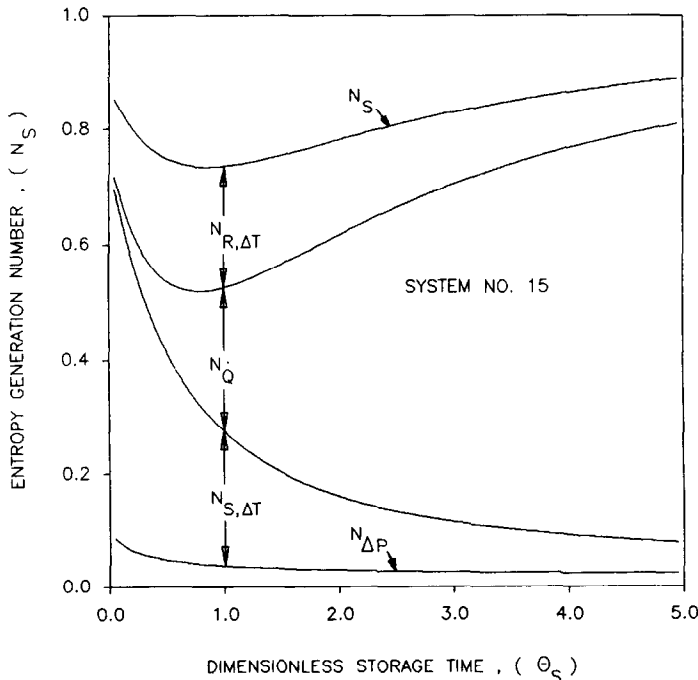


FIG. 4. Effect of storage time on entropy generation by each source of irreversibility in the system.

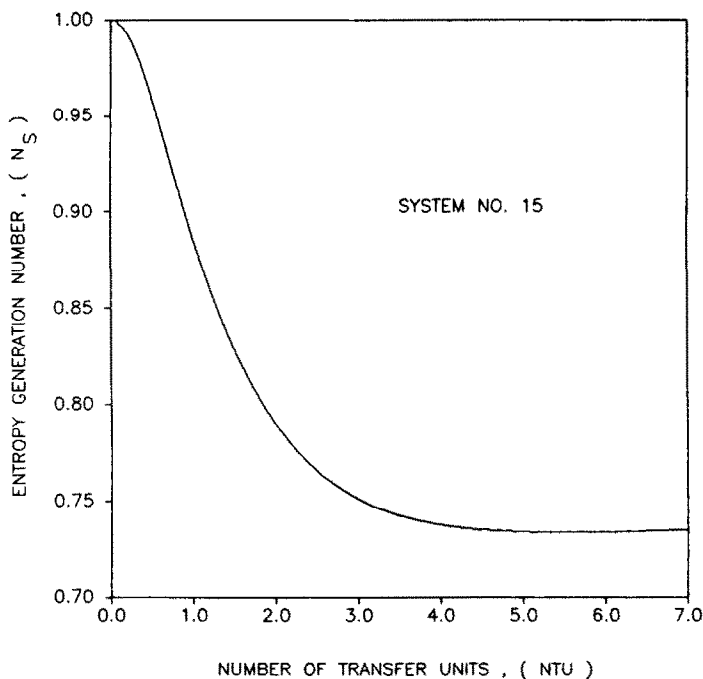


FIG. 5. Effect of heat exchanger NTU on entropy generation number for an otherwise optimal system.

exiting stream of hot gas and the surroundings during the storage process, which is portrayed by N_Q , becomes the dominant source of entropy generation. The entropy generated due to heat transfer in the heat exchanger duct during the removal process, which is represented by $N_{R,\Delta T}$, increases with storage time until it reaches a maximum value at $(\theta_S)_{opt}$ and thereafter decreases with increasing storage time. Finally, the viscous effects in the heat exchanger duct, which are given by $N_{\Delta P}$, make a small, but not negligible, contribution to N_S .

The curve formed by the intersection of the entropy generation number surface with the plane for which $\theta_S = (\theta_S)_{opt}$ is given in Fig. 5. This curve clearly indicates that N_S is a very weak function of NTU over a wide range of values near the optimum point, which is of some practical importance, since this shows that the number of transfer units could be reduced from the optimum value of 5.5533 to values as low as 3 without seriously reducing the performance of the system.

As shown in Table 2, the First Law efficiency, η_1 , of the optimized system is only 57.68%. Thus, when designed and operated in a thermodynamically efficient manner, the system stores just over half of the maximum possible amount of thermal energy. The influence of the storage time on the First Law efficiency for an otherwise optimal system is shown in Fig. 6. As expected, the First Law efficiency increases monotonically with storage time. It approaches its limiting value of unity at a dimensionless storage time of approximately 5. Figure 4, however, shows that when $\theta_S = 5$, that the entropy generation number is approximately 0.89; that is, that 89% of the

commodity of real value, the thermodynamic availability, is destroyed (the Second Law efficiency of the system is only 11%). This discrepancy between the First Law and Second Law performance criteria clearly illustrates the necessity of employing Second Law techniques in order to correctly optimize the thermodynamic efficiency of a thermal energy storage system.

Now that the results for a typical optimum system have been presented, the overall parametric trends of the optimization study will be discussed.

General results of the optimization study

For the ranges of the parameters that were investigated, the dimensionless mass velocity, \bar{G} , had the largest effects of any parameter on system design, operation and performance. As may be seen in Fig. 7, values of \bar{G} that are 'too large' (on the order of 0.5) result in reductions in system performance (increases in N_S). These reductions are caused by greatly increased viscous losses in the heat exchanger duct, which are characterized by the increases in the pressure ratios (P_{IS}/P_0) and (P_{IR}/P_0) with increasing \bar{G} as shown in Tables 1-3. This trend would seem to dictate minimizing \bar{G} , however, as shown in Fig. 8, values of \bar{G} that are 'too small' (on the order of 0.005) result in undesirably large heat exchangers ($NTUs$). Inspection of both Figs. 7 and 8 suggests that a compromise between these two effects is obtained when \bar{G} is selected to be approximately 0.05, which is in agreement with a similar result found by Krane [18] in a study of thermal energy storage systems with Joulean heaters. Thus, all further discussions will be

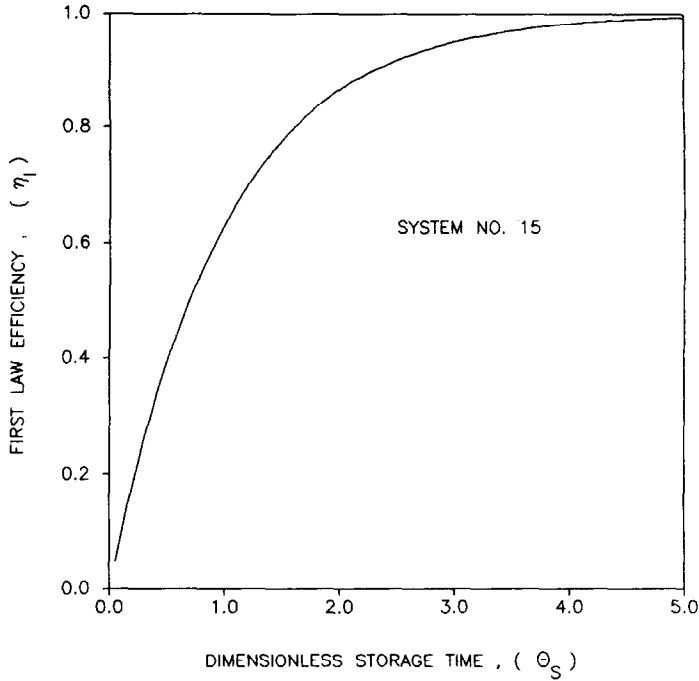


FIG. 6. Effect of storage time on first law efficiency for an otherwise optimal system.

limited to those practically important systems for which $\bar{G} = 0.05$ (systems 4-6, 13-15, 22-24 and 28-33).

Figures 7 and 8 also show that the effects of the parameter ϵ , which characterizes the requirement for the system to have a certain 'tare capacity' in order to deliver thermal energy to the load, and the parameter

(\dot{m}_R/\dot{m}_S) , the ratio of the mass flowrates of gas in the removal and storage processes, on both system performance (N_S) and heat exchanger size (NTU) are relatively small.

Tables 1-5 show that for all systems for which $\bar{G} = 0.05$ that 0.5681 (system 33) $\leq (\theta_s)_{opt} \leq 1.200$ (system 22) and 0.4678 (system 22) $< (NTU)_{opt}$

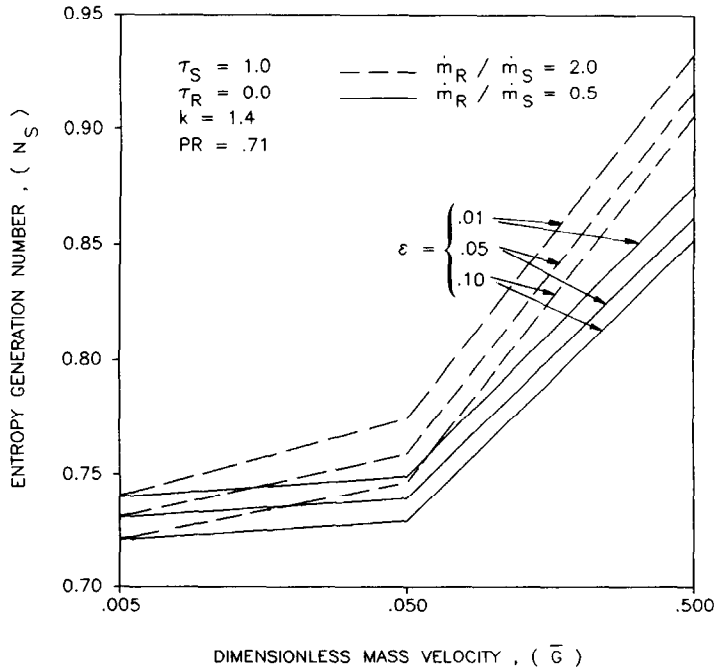


FIG. 7. Effect of the dimensionless mass velocity parameter on system performance.

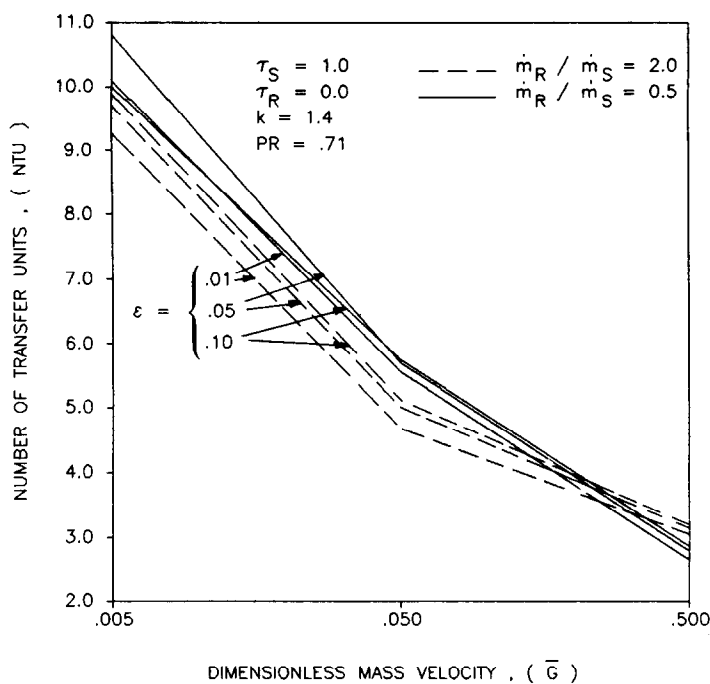


FIG. 8. Effect of the dimensionless mass velocity parameter on heat exchanger NTU .

≤ 6.217 (system 30). Thus, Bejan's observation [8] that the dimensionless storage time should be of order unity for a well-designed system, which he reached by analyzing the storage process alone, seems to hold when the analysis is extended to portray the entire storage-removal cycle. We may also conclude that the heat exchangers required for these systems are all of a reasonable size ($NTU < 6.217$).

As is true for system 15, the thermodynamic efficiencies of all the other practically important ($\bar{G} = 0.05$) systems examined in this study are extremely low. For $\tau_S = 1.0$, the entropy generation number, N_S , varies from 0.7292 for system 6 to 0.7741 for system 3, while for $\tau_S = 2.0$, it varies from 0.6284 for system 33 to 0.7141 for system 28. Thus, since the most efficient optimal system analyzed in the study (system 33) destroys 62.84% of the availability, we may readily conclude that *sensible heat thermal energy storage systems are inherently inefficient devices*. Two aspects of their characteristic inefficiency require further comment. First, as noted in the analysis, the treatment of the storage element as a lumped component prevents the inclusion of an additional source of irreversibility in the analytical model, namely, entropy generation by heat transfer through finite temperature differences *within* the storage element. Thus, if the storage element was modeled more realistically as a distributed component, the resulting entropy generation numbers would be somewhat higher than those calculated in the present study. Second, as shown in Tables 1-5, the entropy generated by heat transfer between the exiting stream of hot gas and the surroundings during the storage process constitutes a

major portion of the total entropy generated during the entire storage-removal cycle. (This contribution, given by the ratio N_Q/N_S , varies from a minimum of 24.4% of the total for system 31 to a maximum of 31% of the total for system 28.) This led Bejan [8] to suggest connecting multiple thermal energy storage systems in series. In such a configuration, the hot gas exiting one unit would enter the next unit instead of mixing with the surroundings. In principle, this would continue until the gas stream exiting a unit during the storage process was essentially at atmospheric temperature. Thus, connecting thermal energy storage systems in series could virtually eliminate, or, at worst, greatly reduce, this contribution to the total entropy generation. The point to be made here, however, is that *when this option is not employed* and the hot gas stream is dumped to the surroundings during the storage process, *this source of entropy generation must be charged to the storage system*.

Comparison of system 15 ($\tau_S = 1.0$, $\tau_R = 0.0$) and system 30 ($\tau_S = 2.0$, $\tau_R = 0.0$) shows that increasing the dimensionless inlet temperature of the hot gas flow in the storage process can marginally increase the thermodynamic performance (decrease the entropy generation number) of a thermal energy storage system. Such increases in performance, however, are achieved at the expense of increased heat exchanger size (here, an increase of approximately 11%). The effect of the dimensionless gas inlet temperature for the removal process, τ_R , on the design and operation of the system can be seen by comparing systems 30 and 33. These systems are identical except that $\tau_R = 0.0$ for system 30 and 0.1 for system 33. As might have been

expected, Tables 4 and 5 show that increasing τ_R from 0.0 to 0.1 decreases the entropy generation number, N_S , from 0.6996 to 0.6284 and reduces the required heat exchanger size (NTU) from 6.217 to 5.864. These results can be used to establish an important requirement for the analysis of thermal energy storage systems, namely, that *the entire storage-removal cycle, not just the storage process alone, must be analyzed in order to optimize the design and operation of such a system.* This follows immediately from the results for systems 30 and 33. All of the parameters which describe the storage processes for these two systems are identical. The systems differ only in a parameter, τ_R , that pertains to the *removal* process. Thus, an optimization scheme which is based solely on an analysis of the storage process would incorrectly predict that the dimensionless storage time, θ_S , the heat exchanger NTU , and the entropy generation number, N_S , should be the same for both systems.

CONCLUSIONS

The present analysis and that of Bejan [9] show that the Second Law of thermodynamics must be used to design thermal energy storage systems with the highest possible thermodynamic efficiencies. Thus, from the viewpoint of Second Law analysis we may conclude that: (1) the real purpose of a thermal energy storage system is not to store energy, but to store thermodynamic availability (exergy); (2) both design and operational parameters must be considered when optimizing the thermodynamic performance of a thermal energy storage system; (3) sensible heat energy storage systems are inherently inefficient devices; and (4) an entire storage-removal cycle as opposed to the storage process alone, must be analyzed in order to optimize system performance.

In closing, we should note that the optimum performance of a thermal energy storage system must ultimately be defined in economic terms. The reader, however, should recall Bejan's definition of an optimal thermal system as 'the least irreversible system that the designer can afford' and realize that the present analysis provides the *engineering* tools to design such systems.

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REFERENCES

1. D. R. Glenn, Technical and economic feasibility of thermal energy storage, C00-2558-1, NTIS, Springfield, VA (February 1976).
2. Committee on Advanced Storage Systems, Criteria for energy storage R & D, National Academy of Science, Washington, DC (1976).
3. F. P. Bundy, C. S. Herrick and P. G. Kosky, The status of thermal energy storage, 76CRD041, General Electric Corp., Schenectady, NY (April 1976).
4. G. Beckman and P. V. Gilli, *Thermal Energy Storage*, Chap. 3, Springer-Verlag, New York (1984).
5. J. Jensen and B. Sorensen, *Fundamentals of Energy Storage*, Chap. 1, John Wiley, New York (1984).
6. D. C. Golibersuch *et al.*, Thermal energy storage for utility applications, 75CRD256, General Electric Corp., Schenectady, NY (December 1975).
7. F. W. Schmidt and A. J. Willmott, *Thermal Energy Storage and Regeneration*, Chaps 2-4, Hemisphere, Washington, DC (1981).
8. A. Bejan, *Entropy Generation Through Heat and Fluid Flow*, John Wiley, New York (1982).
9. A. Bejan, Two thermodynamic optima in the design of sensible heat units for energy storage, *J. Heat Transfer* **100**, 708-712 (1978).
10. M. R. von Spakovsky and R. B. Evans, Detailed second law design of components in complex thermal systems. In *Second Law Aspects of Thermal Design* (Edited by A. Bejan and R. L. Reid), ASME HTD-Vol. 33, pp. 27-37. ASME, New York (1984).
11. R. B. Evans *et al.*, Essergetic analysis for process design and synthesis. In *Efficiency and Costing* (Edited by R. A. Gaggioli), pp. 239-262. American Chemical Society, Washington, DC (1983).
12. C. A. Frangopoulos, Thermoeconomic functional analysis: a method for optimal design or improvement of thermal systems. Ph.D. thesis, Georgia Institute of Technology, Atlanta, GA (1983).
13. M. J. Moran, *Availability Analysis: a Guide to Efficient Energy Use*, p. 86. Prentice-Hall, Englewood Cliffs, NJ (1981).
14. A. Bejan, Second law analysis in heat transfer and thermal design. In *Advances in Heat Transfer*, Vol. 15, pp. 1-58. Academic Press, New York (1982).
15. J. H. Lienhard, *A Heat Transfer Textbook*, p. 321. Prentice-Hall, Englewood Cliffs, NJ (1981).
16. L. S. Lasdon, A. D. Waren and M. W. Ratner, GRG2 user's guide, CIS-T8-01, Computer and Information Science Department, Cleveland State University, Cleveland, OH (March 1978).
17. L. S. Lasdon and A. D. Waren, Generalized reduced gradient software for linearly and nonlinearly constrained problems, WP77-85, Bureau of Business Research, University of Texas at Austin, Austin, TX (October 1977).
18. R. J. Krane, A second law analysis of a thermal energy storage system with joulean heating of the storage element, ASME Paper 85-WA/HT-19, 1985 Winter Annual Meeting of the American Society of Mechanical Engineers, Miami, FL (November 1985).

ANALYSE SELON LA SECONDE LOI, DE L'OPTIMISATION DES SYSTEMES DE STOCKAGE D'ENERGIE THERMIQUE

Résumé—Des techniques d'analyse, selon la seconde loi, basées sur la minimisation de l'entropie créée, sont appliquées à l'optimisation du dimensionnement et de l'opération d'un système de stockage de chaleur sensible dans lequel l'élément accumulateur est à la fois chauffé et refroidi par des écoulements de gaz. Les résultats de cette étude montrent: (1) un cycle opératoire complet, accumulation et décharge, peut être considéré (en opposition avec l'accumulation seule) pour optimiser le fonctionnement et le dimensionnement du système; et (2) un système typique optimal détruit approximativement 70-90% de ce qui entre et, par suite, il y a un rendement thermodynamique extrêmement faible.

EXERGETISCHE ANALYSE VON WÄRMESPEICHERSYSTEMEN

Zusammenfassung—Auslegung und Betriebsweise eines Wärmespeichers für fühlbare Wärme, der durch einen Gasstrom beheizt und gekühlt wird, werden im Hinblick auf den zweiten Hauptsatz optimiert, wobei die Minimierung der Entropieerzeugung als Kriterium dient. Die Ergebnisse der Untersuchung zeigen, daß ein vollständiger Betriebszyklus, bestehend aus Beladung und Entladung, betrachtet werden muß (und nicht allein der Beladevorgang), um Auslegung und Betriebsweise eines solchen Systems zu optimieren. Sie zeigen weiter, daß ein typisches optimales System etwa 70–90% der einströmenden Exergie vernichtet und damit einen sehr geringen thermodynamischen Wirkungsgrad besitzt.

ПРИМЕНЕНИЕ ВТОРОГО ЗАКОНА ТЕРМОДИНАМИКИ ДЛЯ АНАЛИЗА
ОПТИМАЛЬНОЙ КОНСТРУКЦИИ И РЕЖИМА РАБОТЫ СИСТЕМ
АККУМУЛИРОВАНИЯ ТЕПЛОВОЙ ЭНЕРГИИ

Аннотация—Методика оптимизации на основе второго начала, базирующаяся на минимизации производства энтропии, используется для анализа конструкций и режимов работы системы аккумуляции тепловой энергии, в которой аккумулярующий элемент как нагревается, так и охлаждается потоками газов. В результате исследования установлено: (1) для оптимизации конструкции и режимов работы системы необходимо рассматривать полный рабочий цикл, состоящий как из процесса аккумуляции, так и отвода тепла, а не только процесс аккумуляции, (2) в типичной оптимизированной системе теряется около 70–90% подводимой энергии, и следовательно она имеет чрезвычайно низкий термодинамический коэффициент полезного действия.